

WUFI Passes Benchmark Test of EN 15026

WUFI complies with the general requirements of standard EN 15026 and passes its benchmark test:

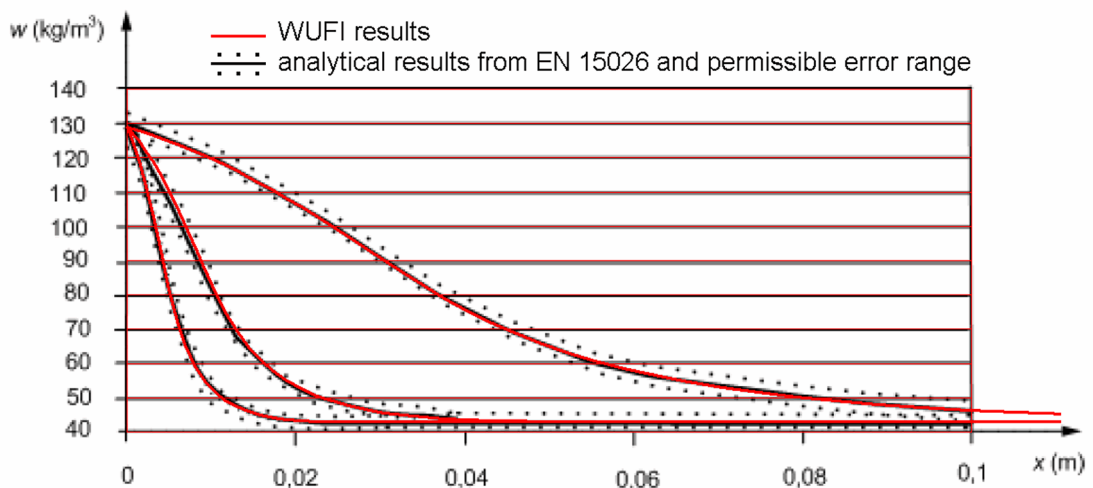


Figure A.1 — The moisture distribution at 7 days, 30 days and 365 days
(background image: EN 15026)

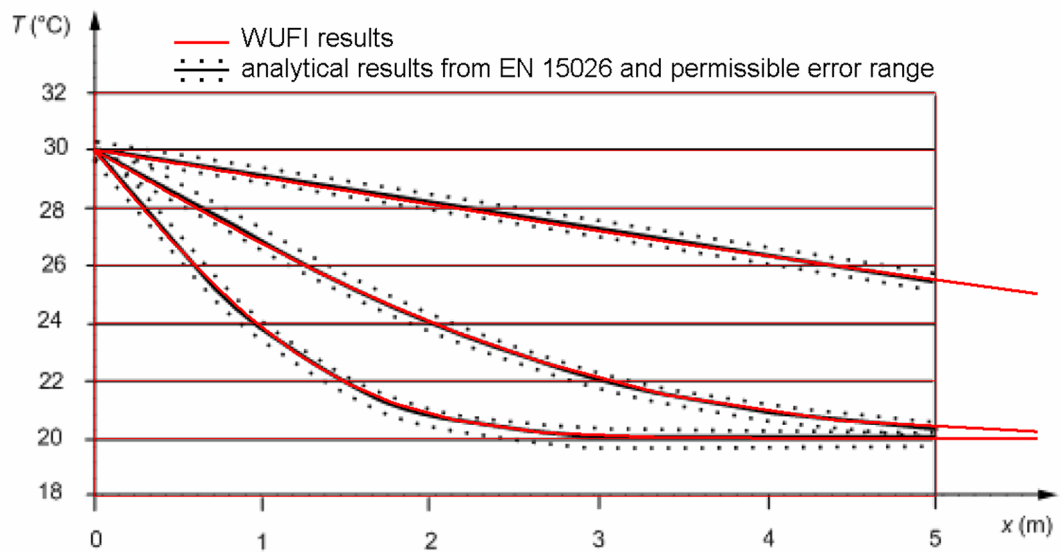


Figure A.2 — The temperature distribution at 7 days, 30 days and 365 days
(background image: EN 15026)

The benchmark test of standard EN 15026 simulates the coupled thermal and hygric behavior of a homogeneous semi-infinite specimen which originally is in equilibrium with a constant surrounding climate ($\vartheta = 20\text{ }^{\circ}\text{C}$, $\varphi = 50\%$) and then is exposed to a climatic step change to $\vartheta = 30\text{ }^{\circ}\text{C}$, $\varphi = 95\%$. The sudden increase of temperature and equilibrium water content at the surface causes heat and moisture fluxes to flow into the interior of the material. The benchmark requires the software under test to calculate the temperature and water content profiles resulting after 7, 30 and 365 days. These profiles must not deviate by more than 2,5 % from the reference solution provided by the standard.

Material properties

The standard specifies the hygrothermal material properties to be used for the specimen. The moisture-dependent properties are provided as analytical functions which need to be tabulated for input to WUFI. In addition, the vapour permeability and the liquid conductivity must be converted to the equivalent quantities used by WUFI's transport model.

Basic properties

The volumetric heat capacity (i.e. per unit volume) of the dry material is specified as

$$\rho_0 \cdot c_0 = 1.824 \cdot 10^6 \text{ J/m}^3\text{K}.$$

WUFI needs the bulk density ρ_0 and the specific heat capacity (i.e. per unit mass) c_0 as separate input quantities. The number specified by the standard may be arbitrarily factored into components ρ_0 and c_0 as long as the product $\rho_0 \cdot c_0$ remains the same. The arbitrary choice

$$c_0 = 850 \text{ J/(kg K)} \text{ (typical value for mineral materials)}$$

results in

$$\rho_0 = 2146 \text{ kg/m}^3.$$

The standard requires the porosity of the material to be chosen so that the maximum water content is equal to the maximum value of the moisture storage function ($u_f = 146 \text{ kg/m}^3$, see below). Thus,

$$\text{porosity} = 146 \text{ kg/m}^3 / 1000 \text{ kg/m}^3 = 0.146.$$

Thermal conductivity is specified as a moisture-dependent function:

$$\lambda = 1.5 + \frac{15.8}{1000} \cdot w$$

This function depends linearly on the water content w ; it is therefore sufficient to enter the two extremal values of the thermal conductivity

$$\begin{aligned} \lambda(w = 0 \text{ kg/m}^3) &= 1.5 \text{ W/mK and} \\ \lambda(w = 146 \text{ kg/m}^3) &= 1.5 + 15.8 / 1000 \cdot 146 = 3.8068 \text{ W/mK} \end{aligned}$$

in WUFI's thermal conductivity table. WUFI automatically uses linear interpolation to find intermediate values and thus reproduces the required linear function.

Moisture storage function

The moisture storage function of a material describes the equilibrium amount of water to which the moisture content in the material will adjust for a given suction pressure p_{suc} in the capillary pore water or for a given relative humidity φ of the pore air. Suction pressure and relative humidity can be converted into each other by means of the Kelvin equation and may thus be used equivalently:

$$p_{\text{suc}} = -R_{\text{H}_2\text{O}} \cdot T \cdot \rho_w \cdot \ln(\varphi)$$

p_{suc} : capillary suction pressure, Pa

ρ_w : density of water, 1000 kg/m^3

$R_{\text{H}_2\text{O}}$ = R/M_w : gas constant for water vapor, $\text{J}/(\text{kg K})$

R : universal gas constant, $8.314 \text{ J}/(\text{mol K})$

M_w : molecular weight of water, 0.018 kg/mol

T : temperature, K
 φ : relative humidity, -

The standard gives the moisture storage function for the benchmark material as analytical formulas, expressed optionally as a function of the capillary suction pressure or as a function of the relative humidity:

$$w = \frac{146}{(1 + (8 \cdot 10^{-8} \cdot p_{\text{suc}})^{1.6})^{0.375}}$$

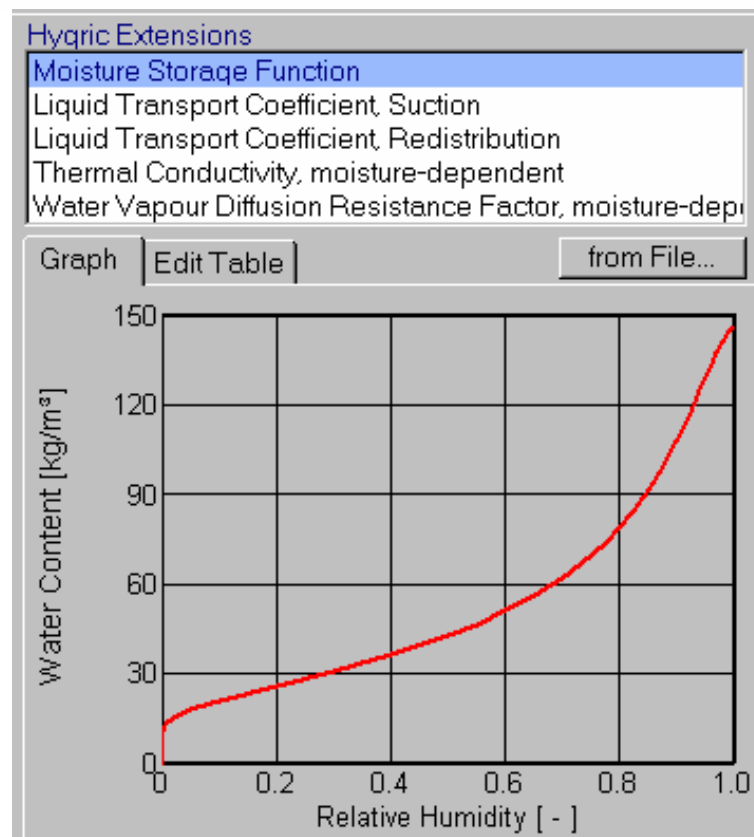
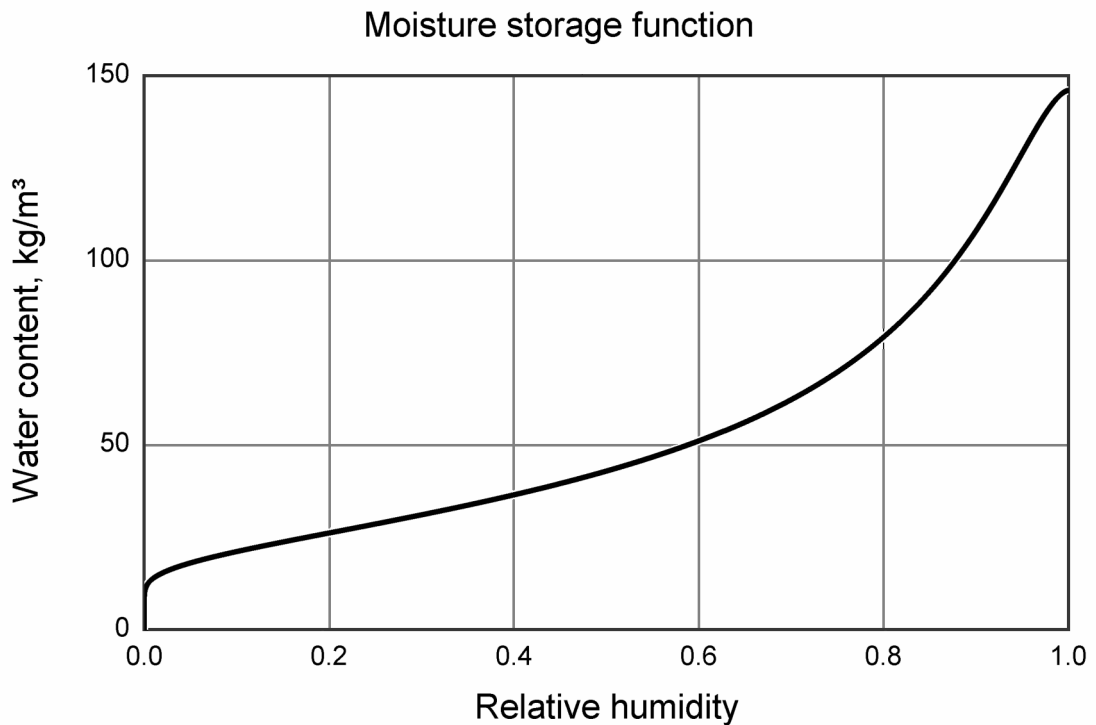
$$w = \frac{146}{(1 + (-8 \cdot 10^{-8} \cdot R_{\text{H}_2\text{O}} \cdot T \cdot \rho_w \cdot \ln(\varphi))^{1.6})^{0.375}}$$

w : water content, kg/m³
 p_{suc} : capillary suction pressure, Pa
 φ : relative humidity, -
T : reference temperature, 293.15 K

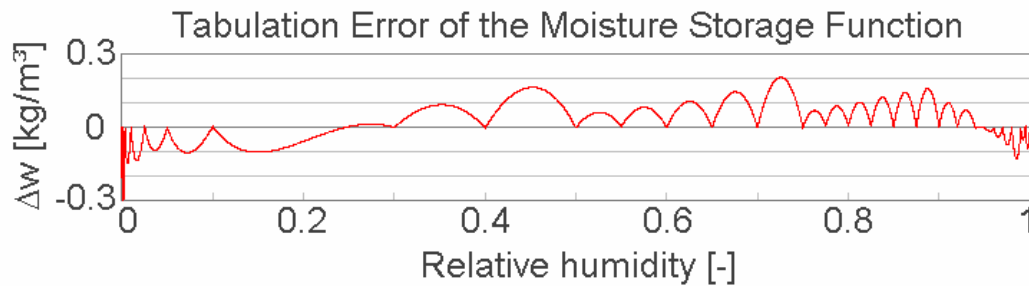
WUFI expects the equilibrium moisture content as a function of relative humidity, so it uses the second alternative. Since WUFI treats the moisture storage function as temperature-independent (a model simplification adopted by the standard, too), the temperature T appearing in this formula is not to be taken as the temperature of the respective grid element during the calculation, but as a predetermined fixed value, in this case the reference temperature specified by the standard, 293.15 K.

The function given by the standard has to be entered in WUFI as a table of discrete function values. WUFI interpolates linearly between the table entries; therefore the tabulation points must be chosen in such a way that the differences between the interpolated function and the original function remain sufficiently small. In particular, strongly curved regions of the function must be covered by a sufficient number of tabulation points. Also, in very steep regions of a function, a comparison of the original and the interpolated curves purely by eye may be misleading, since two curves running close to one another may appear to agree well, while in reality the crucial distance in y-

direction may be unacceptable. These remarks apply to all curves which need to be tabulated; the present moisture storage function is quite unproblematic in both respects.



For the tabulation used here, the tabulation error of the whole function remains below 0.3 kg/m³; for the humidities occurring in the benchmark case (they are limited to values between 50 % and 95 %) it even remains below 0.2 kg/m³.



μ-value

The vapor diffusion flow g_v in the material is proportional to the local gradient $\partial p / \partial x$ of the partial vapor pressure p :

$$g_v = -\delta_p \cdot \frac{\partial p}{\partial x} = -\frac{\delta}{\mu} \cdot \frac{\partial p}{\partial x}$$

The material-dependent proportionality coefficient δ_p must account for all factors affecting the diffusion flow: temperature and pressure of the pore air, ratio of pore volume and total volume, pore radius distribution, pore inter-connectivity, water content etc. This complex behavior can be treated more easily if δ_p is split in two components which separately describe the material-independent influence of the pore air and the material-dependent influence of the pore space structure:

$$\delta_p = \frac{\delta}{\mu},$$

δ_p : vapor permeability of the material, kg/(m s Pa)

δ : vapor permeability of still air, kg/(m s Pa)

μ : vapor diffusion resistance factor of the material, -

WUFI uses this splitting. It only needs the (possibly moisture-dependent) material-specific diffusion resistance factor μ as input. It automatically computes the vapor permeability in the pore air δ from the following formula, taking temperature and pressure of the pore air into account:

$$\delta = \frac{1.968 \cdot 10^{-7} \cdot (\vartheta + 273)^{0.81}}{P}$$

ϑ : air temperature, °C

P : barometric pressure, Pa

Using the internally computed δ and the μ -value entered by the user, WUFI then computes δ_p in order to determine the diffusion flow.

Instead of the μ -value, the standard specifies the complete material-dependent vapor permeability δ_p for the benchmark material:

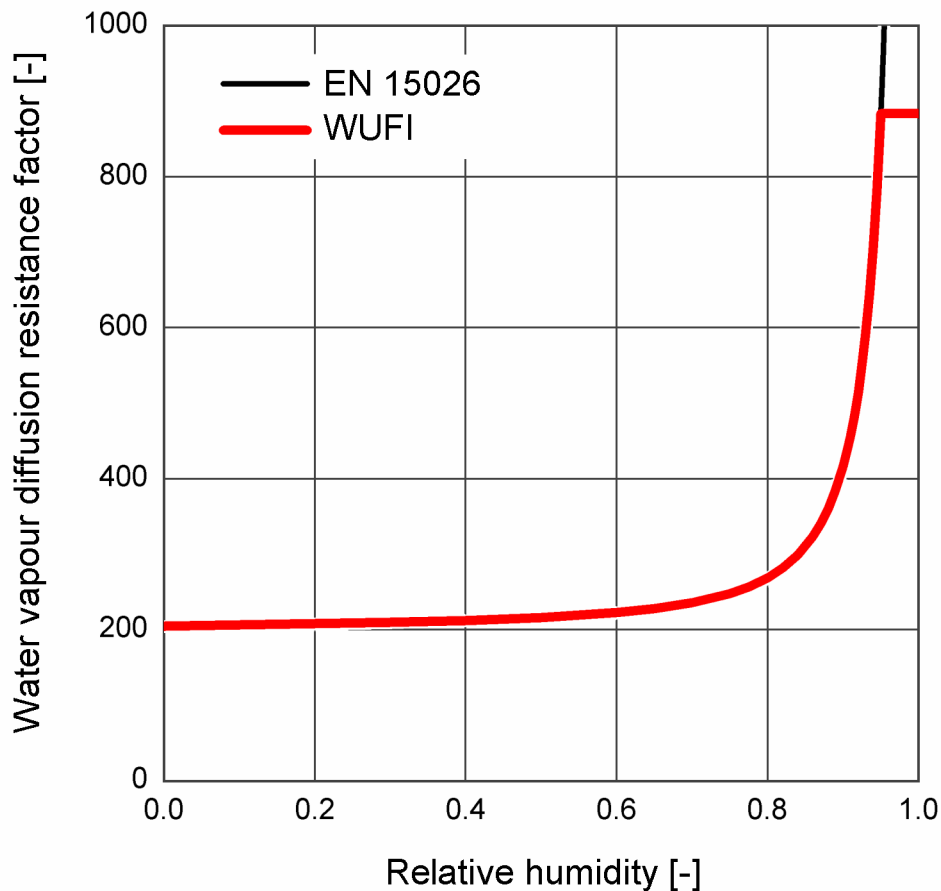
$$\delta_p = \frac{M_w}{RT} \cdot \frac{26.1 \cdot 10^{-6}}{200} \cdot \frac{1 - \frac{w}{146}}{0.503 \left(1 - \frac{w}{146}\right)^2 + 0.497}$$

This formula does not account for temperature and pressure of the pore air, while WUFI automatically does so (the temperature T in the above formula merely serves to convert water vapor concentration to partial pressure).

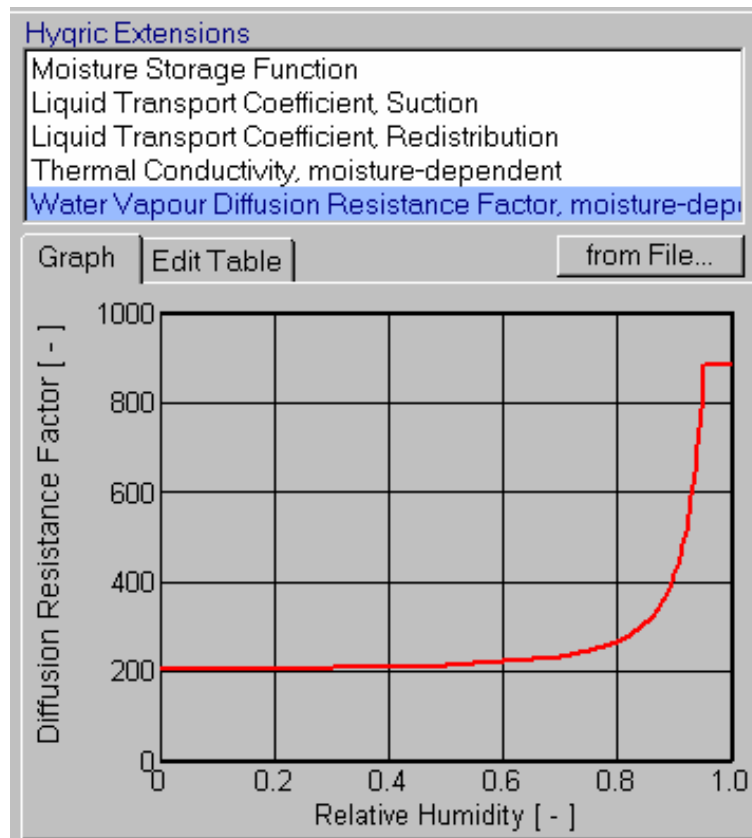
The δ_p specified by the standard has to be converted into the μ -value in such a way that the δ_p internally evaluated by WUFI from the entered μ -value agrees as closely as possible with the required δ_p . Complete agreement cannot be reached since, on the one hand, for the conversion $\delta_p \rightarrow \mu$ a constant δ has to be used (temperature and pressure of the pore air during the experiment being unknown beforehand), and, on the other hand, WUFI's accounting for temperature and pressure during the internal evaluation $\mu \rightarrow \delta_p$ cannot be switched off.

As a compromise, for converting from δ_p to μ one may use estimated values of temperature and pressure which are expected to be representative for the average conditions in the specimen, e.g. $\vartheta = 25 \text{ °C}$ and $P = 101325 \text{ Pa}$:

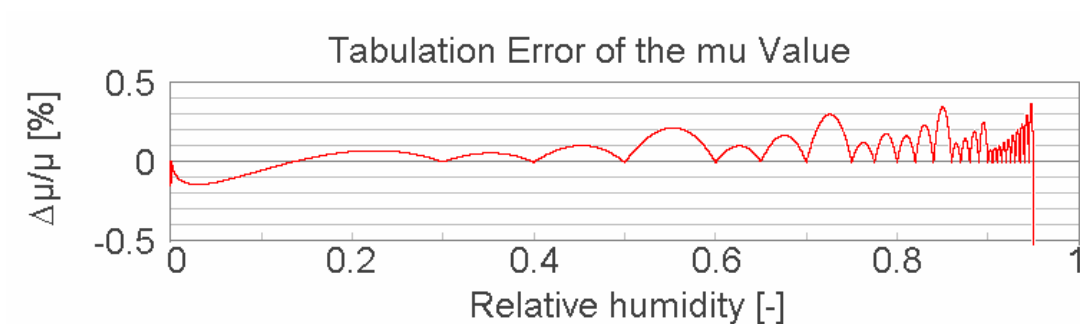
$$\mu = \frac{\delta}{\delta_p} = \frac{1.968 \cdot 10^{-7} \cdot (\vartheta + 273)^{0.81}}{P \cdot \delta_p} = \frac{1.968 \cdot 10^{-7} \cdot (25 + 273)^{0.81}}{101325 \cdot \delta_p}$$



The function $\delta_p(w)$ specified by the standard approaches zero when the material approaches free saturation, the corresponding μ -value therefore approaches infinity. The increasing steepness of the curve would require an impractically fine tabulation, the value $\mu = \text{infinity}$ at free saturation can not be represented in WUFI at all. However, since no humidities above $\varphi = 95 \%$ occur in the benchmark case, it is sufficient for the present purpose to tabulate the function up to 95 % only.



For the tabulation used here, the relative tabulation error in the region between 0 and 95 % remains below 0.4 %.



Liquid transport coefficient

WUFI computes the capillary moisture flow g_w which is caused by the gradient $\partial w / \partial x$ of the water content w by using the equation

$$g_w = -D_w \cdot \frac{\partial w}{\partial x}$$

EN 15026, on the other hand, uses the equation

$$g_w = K \cdot \frac{\partial p_{suc}}{\partial x}$$

(but also approves equivalent equations like the one used by WUFI). The moisture-dependent liquid conductivity K for the benchmark material is given as an analytical function:

$$K = \exp(-39.2619 + 0.0704 \cdot (w-73) - 1.7420 \cdot 10^{-4} \cdot (w-73)^2 - 2.7953 \cdot 10^{-6} \cdot (w-73)^3 - 1.1566 \cdot 10^{-7} \cdot (w-73)^4 + 2.5969 \cdot 10^{-9} \cdot (w-73)^5)$$

The conversion of this function to the liquid transport coefficient D_w used by WUFI is done by means of the equation

$$D_w = -K \cdot \frac{\partial p_{suc}}{\partial w}$$

The function $p_{suc}(w)$ appearing in this equation is the inverse of the moisture storage function $w(p_{suc})$, expressed as a function of suction pressure p_{suc} instead of relative humidity φ (see above). EN 15026 explicitly gives this inverse function as:

$$p_{suc} = 0.125 \cdot 10^8 \cdot \left(\left(\frac{146}{w} \right)^{\frac{1}{0.375}} - 1 \right)^{0.625}$$

Its derivative with respect to w is:

$$\frac{\partial p_{suc}}{\partial w} = -p_{suc} \cdot \frac{0.625}{1 - \left(\frac{146}{w} \right)^{-\frac{1}{0.375}}} \cdot \frac{1}{0.375 \cdot w}$$

EN 15026 allows to neglect the temperature-dependence of liquid transport. Accordingly, it uses the liquid conductivity as given by the above temperature-independent formula for its reference solution.

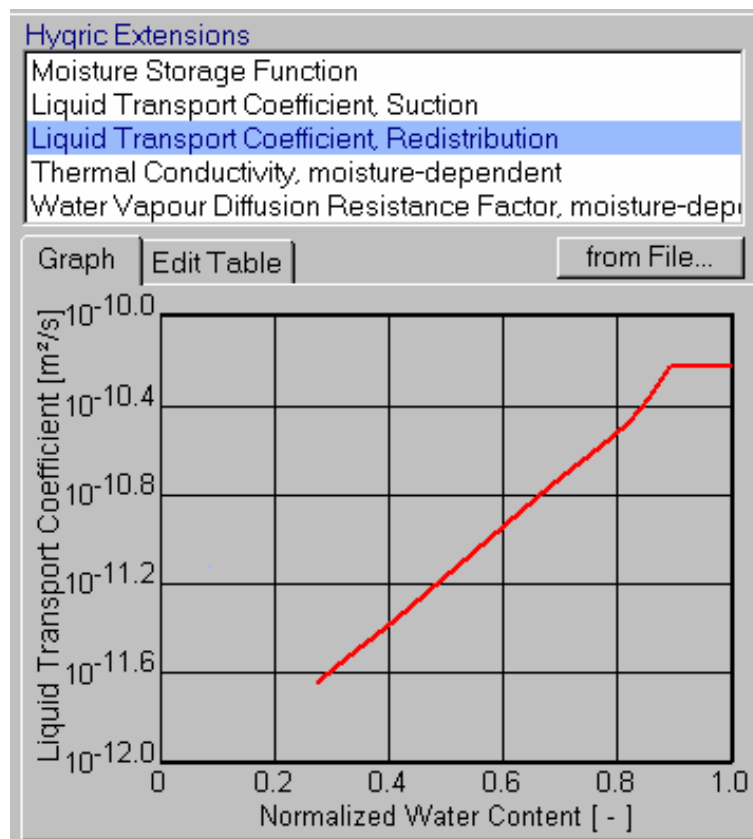
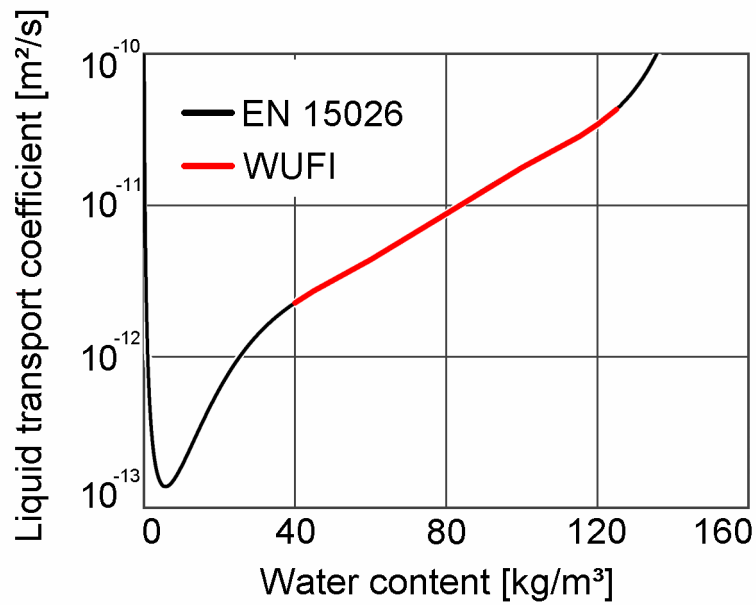
WUFI, however, does account for the temperature dependence of the liquid transport coefficient which is mainly caused by the viscosity of the capillary water decreasing with rising temperature. To this end, WUFI considers the values entered by the user as the reference values valid for 20 °C and corrects them for the actual temperature in each grid element by multiplying them with the viscosity correction factor

$$\text{visfac} = 9.0 \cdot 10^{-5} \cdot \vartheta^2 + 0.0208 \cdot \vartheta + 0.555.$$

In the present case, liquid transport occurs almost exclusively in the region where the temperature has already reached 30 °C, and WUFI thus applies a liquid transport coefficient adjusted to 30 °C wherever any appreciable liquid transport takes place. The original transport coefficient is thus multiplied by the correction factor $\text{visfac}(30 \text{ °C}) = 1.26$. This increase of the transport coefficient increases the amount of absorbed water by the factor $\sqrt{1.26}$, causing the solution to exceed the acceptable deviation from the reference solution (which disregards the temperature dependence). Since the temperature correction of the liquid transport coefficient can not be switched off, the values entered in WUFI must be divided by 1.26 in order to assume exactly the values specified by the standard after the automatic temperature correction has been applied.

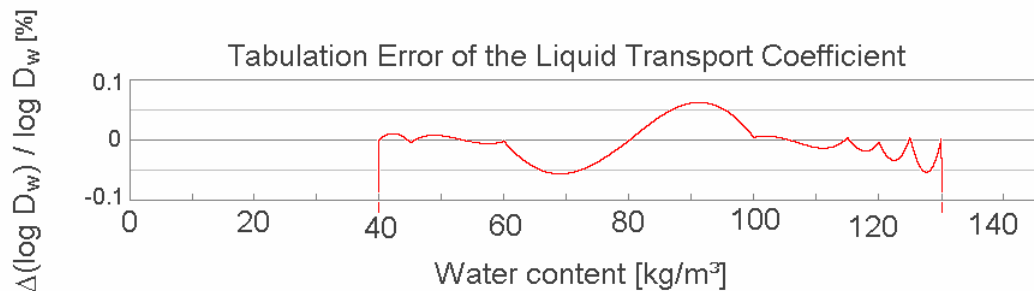
The liquid transport coefficient for WUFI is therefore computed as

$$D_w = -K \cdot \frac{\partial p_{suc}}{\partial w} \cdot \frac{1}{1.26}$$



The analytical function representing the liquid transport coefficient approaches infinity when the water content approaches either zero or free saturation. The increasing steepness at both ends of the curve would require an impractically fine tabulation, the limits $D_w = \text{infinity}$ can not be represented in WUFI at

all. But since in the benchmark case only moisture contents between 42.9 kg/m³ (corresponding to 50 % RH) and 129.0 kg/m³ (corresponding to 95% r.F.) can occur, the tabulation may be confined to moisture contents between 40 and 130 kg/m³. For the tabulation used here, the relative tabulation error of the logarithm of the liquid transport coefficient within this moisture interval remains below 0.1 %.

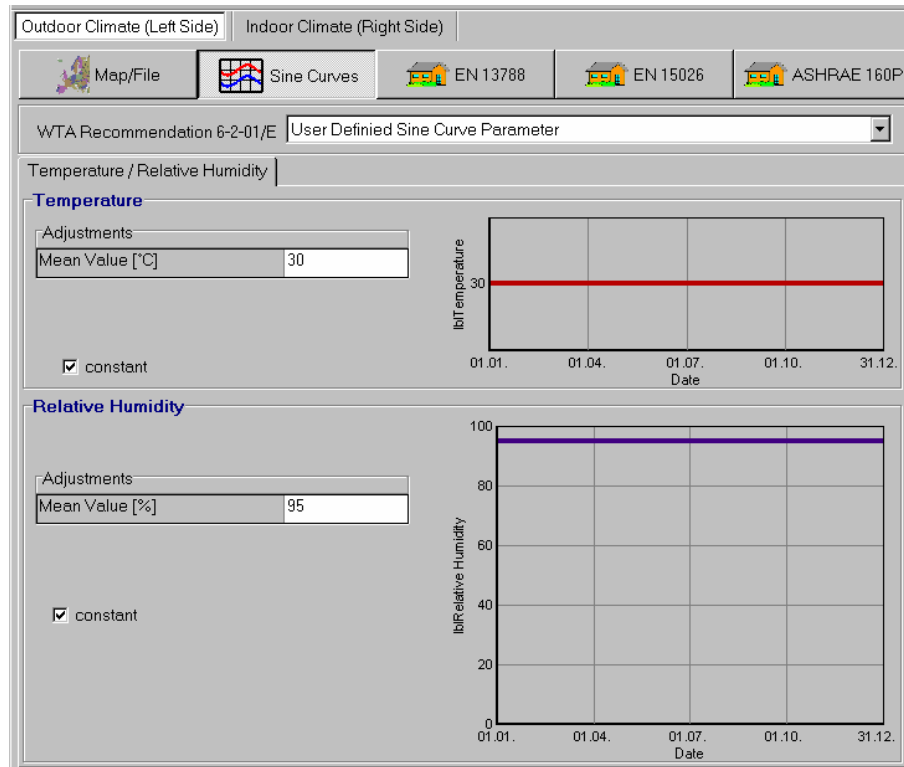


Initial conditions

Initially, the specimen is in equilibrium with the surrounding climate conditions $\vartheta = 20\text{ °C}$ and $\varphi = 50\%$. This hygrothermal state is therefore used as the initial state within the whole specimen.

Boundary conditions

At the start of the calculation, the boundary conditions undergo a step change to $\vartheta = 30\text{ °C}$, $\varphi = 95\%$ and then remain constant during the calculation. In WUFI these new constant boundary conditions can simply be described as sine curves with amplitude zero (option „constant“) and the respective condition as „mean value“.



Surface transfer coefficients

For the free surface of the specimen, the standard specifies that no surface transfer resistances be applied (otherwise an analytical reference solution would not be possible). The heat transfer resistance and the s_d -value for the left surface must therefore be set to zero in WUFI.

At the right, „infinitely distant“ surface, high surface transfer resistances may be applied, so that the boundary conditions acting on that surface have no influence on the specimen (and may be chosen arbitrarily). This also allows to check whether the temperature and moisture changes penetrating into the specimen from the left side can reach the right surface during the calculation.

Case:

Assembly/Monitor Positions | Orientation/Inclination/Height | **Surface Transfer Coeff.** | Initial Conditions

Exterior Surface (Left Side)

Heat Resistance [m²K/W]

wind-dependent includes long-wave radiation parts

Sd-Value [m]

Short-Wave Radiation Absorptivity [-]

Long-Wave Radiation Emissivity [-]

Rain Water Absorption Factor [-]

Interior Surface (Right Side)

Heat Resistance [m²K/W] (User Defined)

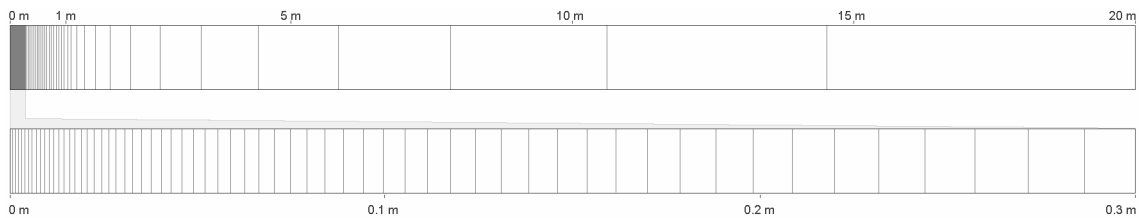
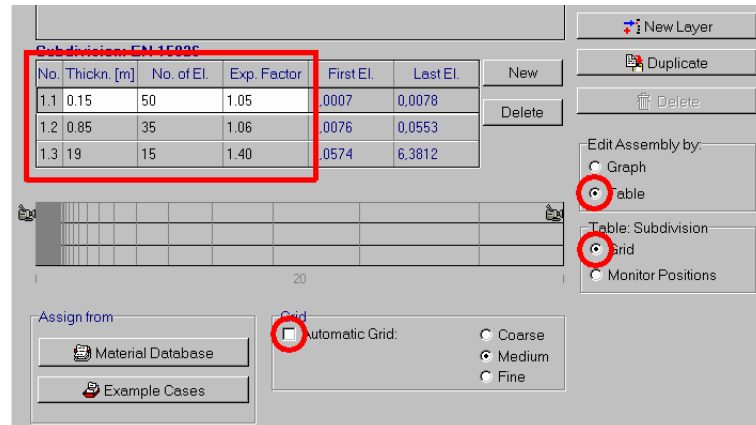
Sd-Value [m]

Numerical grid

The modeled specimen must be thick enough so that during the calculation the temperature and moisture changes do not reach the right surface (assumed to be at an infinite distance) or at least are not affected appreciably. An appropriate thickness can easily be found with a few test calculations. With the component thickness of 20 m adopted here the temperature at the right surface shows only a negligible increase of 0.3 °C during the calculation. The moisture changes remain limited to the leftmost 20 cm anyway.

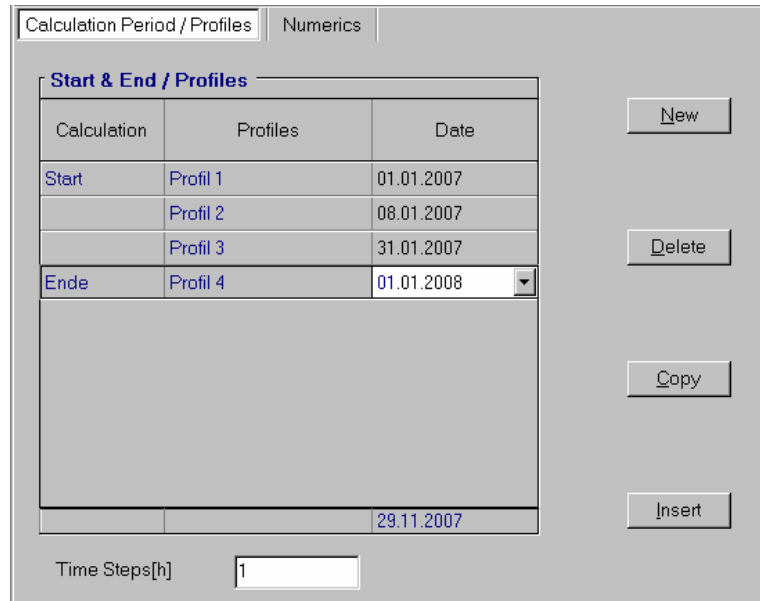
The numerical grid must be able to resolve both the temperature and the moisture profile sufficiently well, although the former extends over the whole thickness while the latter falls off over a very short distance. The present case is too extreme to be handled well by WUFI's automatically generated general-purpose grids, but WUFI provides a manual grid editor which allows to create a custom-made grid for each individual problem. The grid used in the present case distributes the available 100 grid elements by assigning 50 very narrow elements to the leftmost 15 cm of the specimen (where a fine grid is needed to resolve the quickly decreasing moisture profile), 15 wide elements to the rightmost 19 m (for the calculation of the slowly decreasing temperature profile) and the remaining 35 elements to the transition zone in-between. The expansion factors of the sub-grids (1.05, 1.06 und 1.40) are chosen so that the widths of the abutting grid elements at the sub-grid interfaces are not too

different; towards the right side, very wide grid elements are permissible since the temperature profile levels off very smoothly.



Calculation

The calculation period must cover at least 365 days, the starting date is therefore assumed to be 01.01.2007 and the ending date 01.01.2008 (the year numbers are arbitrary and have no meaning for the calculation). In order to obtain the required profiles for the days 7 and 30, the additional dates 08.01.2007 and 31.01.2007 are added to the calculation time table.



Computing the benchmark case only takes a few seconds. With WUFI's ASCII export option the resulting profiles for the various points in time can be written to a text file for matching with the reference solution.

